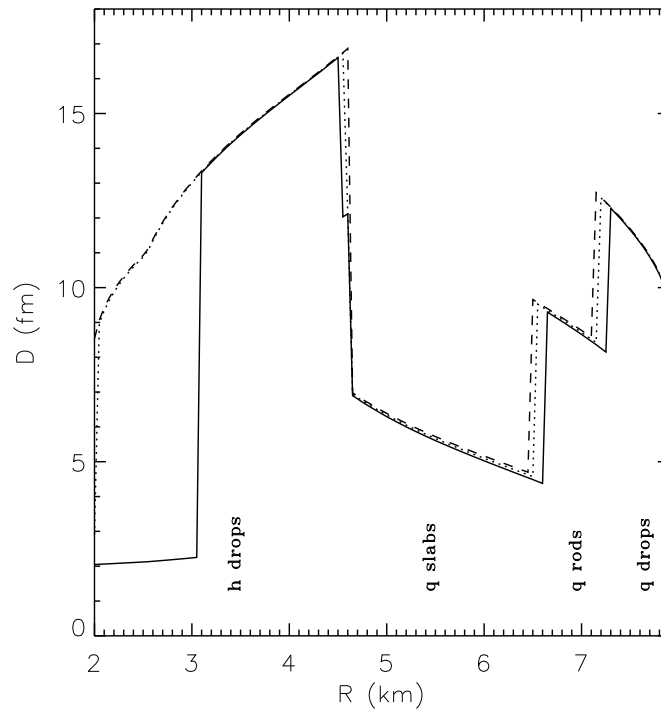


Finite size effects and the mixed quark-hadron phase in neutron stars[†]

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We demonstrate that the form and location (not the size or spacing) of the energetically preferred geometrical structure of the crystalline quark-hadron mixed phase in a neutron star is very sensitive to finite size terms beyond the surface term. We consider two independent approaches of including further finite size terms, namely the multiple reflection expansion of the bag model and an effective surface tension description. Thus care should be taken in any model requiring detailed knowledge of these crystalline structures.

97.60Gb, 97.60Jd, 97.10Cv

I. INTRODUCTION

The possible existence of quark matter in the dense cores of neutron stars was proposed in pioneering work two decades ago [1–3] and has been further studied by a number of authors [4–9]. The theory has recently been reexamined and new insights into the nature of the phase transition were gained which drastically affect the structure of neutron stars [10]. In previous work the mixed or coexistence phase was strictly excluded from the star and a large density discontinuity occurred at the radial point corresponding to the (constant) pressure of the mixed phase. It is now understood that the common pressure and all properties of the two phases in equilibrium vary as their proportion. Consequently the mixed phase can occupy a large radial extent of some kilometers dimension between the two pure phases. Moreover, the mixed phase region is expected to achieve an energy minimum by formation of a crystalline lattice of the rarer phase immersed in the dominant one. The dimensions and geometrical forms of the lattice will vary with the proportion of the phases and hence with depth in the star [10]. The discontinuity in density of the early work is replaced by a region made of an intricate pattern of crystalline forms.

The assumption underlying the previous picture is that the confining transition from quark matter to hadrons is a first order transition with effectively one conserved charge, namely the baryon number. This situation arose in some papers through treatment of the star as being made solely of neutrons (which is beta unstable and therefore an excited state of the star) and in others by the imposition of *local* charge neutrality on a beta stable

system which effectively also reduced the phase transition to one with a single conserved charge [10].

In either case therefore, the transition was made to resemble that of the liquid-vapor transition in water, a constant pressure-temperature one. However first order phase transitions in substances of two or more independent components such as neutron stars in beta equilibrium (for which baryon number and electric charge number are the conserved quantities) behave quite differently [10]. This follows since the conservation laws are *not* local ones but are global. In fact imposition of local neutrality violates Gibbs conditions for phase equilibrium. When the two phases are in equilibrium with each other they can rearrange the concentration of the conserved charges to optimize the total energy while conserving the charges overall. In general, energy will be minimized by a different concentration of conserved charges at each proportion of the phases; hence all properties of the phases also vary with proportion.

In nuclear systems it is the isospin restoring interaction (responsible for the valley of beta stability in nuclei) that exploits the degree of freedom available by the possibility of rearranging concentrations while conserving charges. The consequence is that regions of confined and deconfined matter will have opposite charge densities and a structured Coulomb lattice minimizes the energy. Elsewhere it was found that the unstructured phase would be preferred if the surface tension were too large [11]. We have misgivings about this conclusion however, which we outline next.

In Ref. [11] the surface tension was treated as a free parameter, which it is not. It was therefore possible to find a value that was large enough to put the structured phase at a higher energy than the unstructured one. However, opening a degree of freedom has the effect of lowering energy, or leaving it unchanged, not raising it. Were it possible to do an exact calculation of phase equilibrium including the possibility of spatial structure, without introducing the bulk approximation corrected for Coulomb energy, surface energy, curvature, etc, the above physical principle would be obeyed. It is therefore incorrect in the approximation scheme (bulk+surface+coulomb+curvature) to arbitrarily choose a value of surface tension that places the structured phase above the unstructured one.

The form, size, spacing and location in the star of the crystalline mixed phase was first calculated by Glendenning and Pei [12], where the sum of Coulomb and surface energies was minimized as a function of the proportion of the two phases in equilibrium. Here we extend their calculation to include further finite size effects, i.e. curvature energy and even higher order terms, for the three simple geometries, spheres, rods and slabs. We do this in two completely independent ways, and show that the inclusion of additional finite size effects has a significantly larger influence on the preferred geometrical form, than on the actual size of the form.

In Sec. II we give a general description of the mixed phase. Sec. III and IV are devoted to a description of the two different approaches to include further finite size corrections, and Sec. V contains a general discussion and a summary of our results.

II. THE MIXED PHASE

The equation of state of the confined hadronic phase is calculated as in Ref. [13,14]. The coupling constants of the theory are determined by the bulk nuclear properties, binding energy, $B/A = -16.3$ MeV, saturation density, $\rho_0 = 0.153 \text{ fm}^{-3}$, symmetry energy coefficient, $a_{\text{sym}} = 32.5$ MeV, compression modulus $K = 240$ MeV and effective nucleon mass at saturation, $m^*/m = 0.78$. The ratio of hyperon coupling to mesons as compared to nucleon couplings is chosen in accord with Ref. [14] to be $x_\sigma = 0.6$, $x_\omega = 0.658$.

The bulk properties of the quark phase is described in the bag model at zero temperature with the strong coupling constant, $\alpha_s = 0$. The relevant expressions for the pressure, energy density, baryon number and charge density can be found in Ref. [10]. Throughout, we use a bag constant, $B^{1/4} = 180$ MeV. We take for the quark masses $m_u = 5$ MeV, $m_d = 10$ MeV and $m_s = 150$ MeV. No heavier quarks are present. While the importance on the bulk properties from using non-zero u and d quark masses is negligible, the surface tension is increased about 15 percent in the so-called thin wall limit (only massive quarks contribute to the surface tension).

The relation between the pressure and energy density, and the neutron star's Schwarzschild radius is found from solving the Oppenheimer-Volkoff equations. Moving inwards through the star, the mixed phase becomes energetically favorable when the energy density has increased to a value where the bulk pressures of the two phases have become equal. In our approximation (volume+surface+Coulomb+....) the formation of geometrical structures in the mixed phase only occurs if the sum of finite size and Coulomb energy densities is less than about 10 MeV/fm^3 . In the next section, where finite size corrections are determined from the bag model this does not pose a problem as the sum of energy densities is at

most 7 MeV/fm^3 . In Sec. IV, where the surface energy cannot be explicitly calculated from the model, we ensure that the sum does not exceed 10 MeV/fm^3 by appropriate adjustment of the surface tension. The geometrical structure of the mixed phase occurs to a good approximation against the background of the bulk structure, except in the outermost regions of mixed phase, where the surface pressure from the geometrical structures are comparable with the bulk pressures. The Debye screening lengths were estimated in Refs. [11,15] to be about 5 fm. This is approximately the typical radius of the geometrical structures and thus screening is also only a small effect. We will neglect these minor complications. If the neutron star is massive enough it will have a pure core of quark matter. We focus on a neutron star at the mass limit, which with our choice for the equation of state is at $1.454M_\odot$ and a radius of 10.32 km.

The mixed phase is subdivided into Wigner-Seitz cells, that have total charge zero. Adjacent cells therefore do not interact. Each cell has the same form as the one structure of rare phase it contains. The surrounding region of the other phase is treated as a bulk system. For the simple discrete geometric forms described in [16,12], drops, rods and slabs, each characterized by the dimensionality d equal to 3, 2, and 1, respectively, the Coulomb energy per volume for the three structures can be written as

$$E_C = 2\pi [\rho_h(\chi) - \rho_q(\chi)]^2 r^2 x f_d(x) = C(x)r^2, \quad (1)$$

where ρ_h and ρ_q are the charge densities of the hadronic and quark matter at volume proportion $\chi = V_q/V$ of quark matter. The ratio of structure volume to cell volume is denoted by $x = (r/\mathcal{R})^d$. When quark matter is the rare phase, $x = \chi$, and otherwise, $x = 1 - \chi$. In the case of drops or rods, r is their radius and \mathcal{R} the half distance between centers, whereas for slabs, r is the half thickness. The function $f_d(x)$ is in all three cases given by

$$f_d(x) = \frac{1}{d+2} \left[\frac{1}{d-2} \left(2 - dx^{1-2/d} \right) + x \right]. \quad (2)$$

The preferred geometrical form is found by minimizing the sum of finite size and Coulomb energies. We take two different approaches to get expressions for the finite size corrections beyond the surface term.

III. FINITE SIZE CORRECTIONS IN THE BAG MODEL

From the multiple reflection expansion method [17] used with MIT bag boundary conditions [18], the finite size corrections to the energy can be explicitly calculated. This has been done up to curvature corrections for massless quarks, but the ansatz provided by Madsen [19] for

the curvature term for massive quarks has shown to reproduce exact mode-filling calculations for quarks at zero temperature. The use of MIT bag boundary conditions imply that the thickness of the boundary wall is zero, the so-called thin wall approximation. In this approximation the two phases in equilibrium are treated independently, i.e. any finite size energy contributions from the hadronic phase just add to the quark matter contributions. The surface energy per volume for the quark matter is

$$E_S = \frac{dx\sigma}{r} = \frac{S(x)}{r}, \quad (3)$$

and the curvature energy per volume

$$E_\Gamma = \pm \frac{d(d-1)x\gamma}{r^2} = \pm \frac{\Gamma(x)}{r^2}, \quad (4)$$

where “+” is used when $x = \chi$, and “-” when $x = 1 - \chi$. The curvature term vanishes of course for slabs. Expressions for the surface tension, σ , and curvature coefficient, γ , can be found in Ref. [19]. σ decreases from about 37 to 25 MeV/fm², and γ decreases from 25 to 12 MeV/fm as the chemical potentials drop out through the star. The nuclear liquid drop model give a surface tension $\sigma_{had} \simeq 1$ MeV/fm² for ordinary nuclear matter in vacuum that, by comparison with the quark contribution, can be ignored in the thin wall approximation [20].

When the quark phase is the rare one, the sum of finite size and Coulomb energy densities diverges to plus infinity as a function of r in both the $r \rightarrow 0$ and $r \rightarrow \infty$ limits, and thereby ensuring the existence of a local energy minimum. But the sign change in the curvature energy for a bulk quark phase has devastating consequences, as the sum of energy densities diverges to minus infinity for vanishing r . First we cannot be sure there is a local minimum, and even worse the energy can be lowered by creating more and more smaller and smaller hadronic bubbles in the quark matter. The sign change in the curvature energy is not a pathological defect of the model. The curvature energy is proportional to the mean curvature integrated over the surface of the geometry, and is therefore a signed quantity unlike the surface area. This has explicitly been confirmed by Mardor and Svetitsky [21] in the high temperature, zero chemical potential limit. We take this as a warning of how important it is to ensure that enough terms in the expansion are included.

The next term in a finite size expansion of the energy, at least for massless quarks, is of the form Z_o/r for spheres, and is identified as the Casimir or zero-point energy. Unfortunately, Z_o has not been evaluated in the multiple reflection expansion of the bag model. However, similar procedures, give $Z_o \sim 1$ [22,23]. Bag model fits to the hadronic spectrum are considerably improved if a zero-point term of $-1.84/r$ is added to the energy [24]. About half of it can be explained in terms of center of mass corrections from localization of the hadron [25]. We

therefore choose to treat Z_o as a phenomenological parameter of order one for all three geometries. For the bulk quark phase, where $E_\Gamma < 0$, we will insist upon Z_o being positive; whereas in the bulk hadronic case, where $E_\Gamma > 0$, we will allow both signs for Z_o . Even if Z_o is negative, the sum of finite size and Coulomb energies will not be negative except for radii so small that we do not believe in the expansion anyway. This always ensures the existence of a local energy minimum.

For cylinders and parallel plates the zero-point terms are [26,23] $E_{cyl} = Z_o l/r^2$ and $E_{plate} = Z_o l^2/a^3$, respectively. l is the length of the structure, r the cylinder radius, and a the distance between the plates, in our notation $2r$. The zero-point energies per volume are then

$$E_Z = \frac{Z(x)}{r^4} = \frac{Z_o x}{r^4} \times \begin{cases} \frac{1}{16}, & d = 1, \\ \frac{1}{\pi}, & d = 2, \\ \frac{3}{4\pi}, & d = 3. \end{cases} \quad (5)$$

The sum of finite size and Coulomb energies can be written as

$$E = C(x)r^2 + \frac{S(x)}{r} \pm \frac{\Gamma(x)}{r^2} + \frac{Z(x)}{r^4}. \quad (6)$$

The minimization procedure is carried out numerically for the three geometries in consideration, under the constraint that $2E_C = E_S + 2E_\Gamma + 4E_Z$.

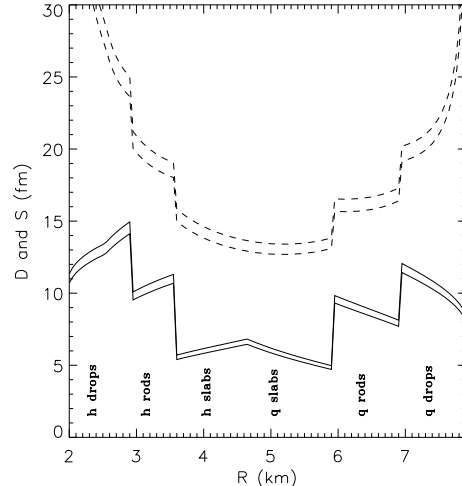


FIG. 1. Mixed phase region in a neutron star showing the diameter D (solid curves) and spacing S (dashed curves) as a function of the radial Schwarzschild coordinate. The curves on top are for massive u and d quarks, in the bottom ones they are assumed massless. “h drops” denotes hadronic drops immersed in bulk quark matter, and so forth.

We plot, as a function of the radial Schwarzschild coordinate, R , the diameters of the different geometric structures present in the mixed phase region of the neutron

star, and the spacing between adjacent structures immersed in the bulk phase. The core of the neutron star up to $R = 2.0$ km consist of pure quark matter, while beyond $R = 7.9$ km the neutron star is composed of ordinary hadronic matter. In Fig. 1 we have neglected the curvature and zero-point energies and show the increase in structure size of about 5%, when the u and d quarks are not assumed massless. All six geometric forms are present in this plot. A new form becomes energetically favorable at each non-differential point on the diameter curve. The discontinuities are only present because of the discrete geometries we treat. Their location is independent of the surface tension, only the radii of the geometries are affected. The radii lie between 2.5 and 7.5 fm, and therefore screening effects are only of minor importance even for the largest structures.

Fig. 2 is a plot similar to Fig. 1 without the spacing curves. Included is now the curvature energy and three zero-point energies, assumed positive in both bulk phases. In comparison with Fig. 1 two striking new features emerge. First, the solid diameter curve, that represents the smallest zero-point energy, $Z_o = 1$, abruptly drops at $R \simeq 3$ km, but without changing the preferred geometry, hadronic drops. The reason is that a second minimum in the sum of finite size and Coulomb energies, Eq. (6), show up at around $r = 1$ fm, if the zero-point term is small enough.

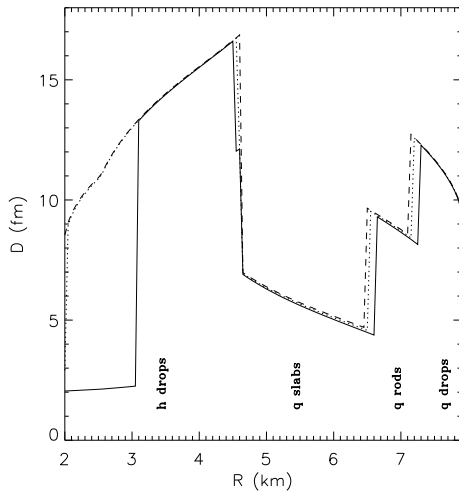


FIG. 2. Similar to Fig. 1 but without the spacing curves. The solid, dotted and dashed curves are for Z_o equal to 1, 1.5 and 2, respectively. Z_o is assumed positive in both bulk phases.

For increasing Z_o the second minimum will eventually not be the global one, this happens when $Z_o > 1.5$, and will finally even disappear. Even though the sum of energies is positive for $Z_o = 1$ as it should be, we neither expect nor believe the model to be valid for such small radii. So we must take $Z_o > 1.5$. Second, no hadronic

slabs are present. This is no big surprise, since the curvature energy is zero for the slabs, while negative for hadronic drops and rods. Hadronic rods are only preferred in a very narrow range when Z_o is unacceptably small. Thus the inclusion of further finite size corrections has large implications for the location, and even presence of the geometric structures in both bulk phases; whereas the sizes are unnoticeably affected, except for small Z_o , where we do not believe in the model.

In Fig. 3, which is similar to Fig. 2, we have fixed the zero-point energy for the bulk quark phase and show how the location and size of the preferred geometry changes for three different values of Z_o for the bulk hadronic phase [27].

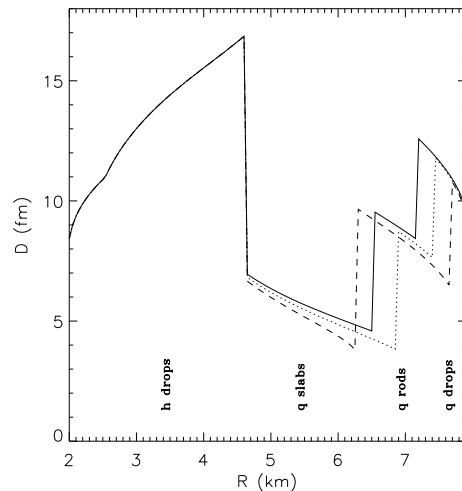


FIG. 3. Similar to Fig. 2. The solid, dotted and dashed curves are for Z_o equal to 1.6, 0 and -1.6 in the bulk hadronic phase. $Z_o = 1.6$ in the bulk quark phase.

For the bulk quark phase the three curves are of course identical. Again we see that the structure size is relatively less affected by the change in Z_o than the location of the geometry. The radial extent of the quark drops decrease with decreasing Z_o . The division between quark slabs and rods does not exhibit the same simple behaviour.

We have assumed that Z_o for all three geometries are identical, at least in each of the two phases. We have no reason to expect this should be correct, but the assumption reduces the possible six different Z_o 's to two, still enough to show the sensitivity of the location of a geometrical structure to the parameter choice. If the small contribution from the hadronic phase is added to the surface energy, only a 3-4% increase in the surface tension, and the outcome plotted in Fig. 3, the only noticeable difference is that hadronic rods are now preferred in a narrow range between hadronic drops and quark slabs, the radii are not visibly affected. Furthermore, the smallest acceptable value for Z_o is reduced to 1.3. Fig. 4 shows

the reason for this sensitivity. In this figure the sum of finite size and Coulomb energies have been plotted as a function of the radial coordinate for each of the three geometries. The discontinuity at $R = 4.6$ km is solely due to the sign change in the curvature energy. The zero-point term is positive in both bulk phases. Notice especially how close the rod and drop curves lie in most regions. It should be obvious that even small changes in parameters could displace the locations of the preferred geometries.

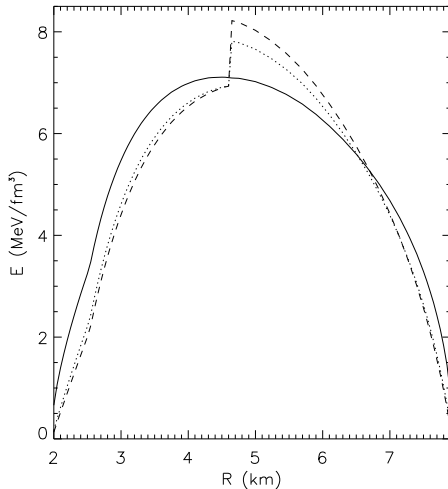


FIG. 4. The sum of finite size and Coulomb energies as a function of Schwarzschild coordinate for $Z_o = 1.6$ in both bulk phases. The solid, dotted, and dashed curves represent slabs, rods, and drops, respectively.

IV. THE MIXED PHASE IN AN EFFECTIVE SURFACE TENSION DESCRIPTION

In connection with the cosmological phase transition, Kajantie *et al.* [28] have derived an expression for the effective surface tension for a diffuse interface between quark and hadronic matter in equilibrium, based on a simple Taylor expansion of the surface tension of a planar surface. Also in the high temperature regime the curvature term is negative in the bag model for hadronic bubbles in bulk quark matter [21]. The sign and magnitude has been confirmed by lattice QCD calculations [29]. This inevitably leads to a negative local minimum in the free energy even above the phase transition temperature. The model of Kajantie *et al.* has the attracting feature of reproducing the negative curvature term when the expression for the effective surface tension is expanded, but still higher order terms prevent the puzzling negative local energy minimum from ever forming.

We cannot immediately use the results for the effective surface tension presented in that paper. The reason

being that in our case the pressures and the surface tension are functions of two variables, namely the baryon chemical potential, μ_n , and the electron chemical potential, μ_e , instead of being just a function of temperature. This reflects the presence of two conserved charges in our case. But because the electron distribution is uniform and $\mu_e \ll \mu_n$, the effective surface tension can still be written in the form, generalized to also include cylindrical and planar geometries (see Appendix A),

$$\sigma(r) \simeq \frac{\sigma(\infty)}{1 \pm (d-1)\delta/r}, \quad \frac{(d-1)\delta}{r} \ll 1, \quad (7)$$

where “+” is used when $x = 1 - \chi$ and “−” when $x = \chi$. d is the dimensionality, $\sigma(\infty)$ the surface tension of a planar surface, and δ is a parameter of dimension length given as

$$\delta = \frac{(\partial\sigma/\partial\mu_n)_{\mu_n^c, \mu_e^c}}{n_{n,q} - n_{n,h}}, \quad (8)$$

where $n_{n,i}$ is the baryon number density for each of the two phases.

We take, as in Ref. [12], for the surface tension

$$\sigma(\chi) = \text{const} \times [\epsilon_q(\chi) - \epsilon_h(\chi)] L, \quad (9)$$

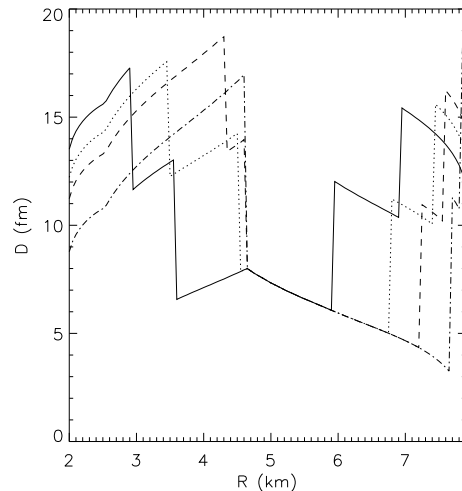


FIG. 5. Similar to Fig. 2. The curves are for δ equal to 0 (solid), 0.5 (dotted), 0.9 (dashed), and 2 fm (dash-dotted), respectively.

where ϵ_i is the energy density for each of the two phases. L is a length scale of the order of the interface thickness, about 1 fm. The constant is chosen so that the sum of finite size and Coulomb energy densities does not exceed 10 MeV/fm³. With this choice for the surface tension (σ varies between 51 and 66 MeV/fm³), δ can actually be calculated, but for our purpose of demonstrating the large sensitivity of the location of a geometrical

structure to the parameter choice, we will treat δ as a free parameter. Estimates based on Eq. (8) show that $\delta(\chi)$ varies between a fraction of one fermi to a few fermi's, but in some regions around $\chi \simeq 0.5$ it may actually become negative.

We plot in Fig. 5 the structure diameter as a function of the radial coordinate for four different values of δ , where Eq. (7) is valid. The solid curve where $\delta = 0$, i.e. no finite size terms beyond the surface term, is similar to the solid curves in Fig. 1. The maximum structure radii are somewhat larger than in the bag model, because of the larger surface tension. When δ is increased, the hadronic drops will take up an increasing fraction of the bulk quark mixed phase region at the expense of hadronic rods and slabs, both of which finally even disappears within the range of the chosen δ 's. For the bulk hadronic matter it is the quark slabs that take up an increasing fraction at the expense of quark rods and drops. If δ is negative, the general tendencies become inverted, quark drops and hadronic slabs will eventually dominate the mixed phase region.

Also in this model the structure sizes are relatively less affected, than the location and even presence of a geometric structure. This is expected because the general trends are of course identical in the two approaches of including further finite size corrections, as the leading contribution comes from the curvature term, which are similar in the two approaches. The curvature term calculated from the bag model is reproduced for $\delta \approx 0.6$, but of course additional terms are different in the two approaches.

V. CONCLUSION

We have used two different models to include finite size corrections beyond the surface term in the sum of finite size and Coulomb energies. Within the multiple reflection expansion of the MIT bag model the surface tension and curvature coefficient can be explicitly calculated. Because of the negative curvature term, present when hadronic matter is immersed in bulk quark matter, we were forced to add an additional term, which was identified as a zero-point energy stemming at least partly from Casimir effects. The zero-point term was treated as a phenomenological parameter, since terms beyond the curvature term have not been determined from the expansion method.

The effective surface tension description was originally developed for the cosmological quark-hadron transition, where a negative curvature term is also present. In the present paper this approach was generalized to the zero temperature, high baryon and electron chemical potential limit. For both models the conclusions are qualitatively the same. It is the form and location, that relative

to the size and spacing, become mostly affected by inclusion of additional finite size terms. The reason is, as shown in Fig. 4, that the minimized sum of finite size and Coulomb energies differ very little between the different geometries, especially between rod and drop like structures. Therefore even small changes in the finite size terms may significantly displace the location and even presence of a given geometry.

This large sensitivity make calculations involving a detailed understanding of the crystalline mixed phase unreliable at present, even for the three simple geometries. But it also tells us that the wide range of glitch phenomena observed in pulsars may be closely related to restructuring in the solid crystalline region of the pulsar.

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APPENDIX A: THE EFFECTIVE SURFACE TENSION

In contrast to the cosmological transition studied in Ref. [28] the pressures of the two phases and the surface tension are now functions of two variables, namely the baryon chemical potential μ_n , and the electron chemical potential μ_e .

By μ_n^c , μ_e^c and P_c we denote the values of the chemical potentials and pressure when the two phases are in equilibrium. In the proximity of the transition, the hadron and quark pressures can be written as

$$P_h(\mu_n, \mu_e) = P_c - n_{n,h}(\mu_n^c - \mu_n) - n_{e,h}(\mu_e^c - \mu_e) \quad (A1)$$

and

$$P_q(\mu_n, \mu_e) = P_c - n_{n,q}(\mu_n^c - \mu_n) - n_{e,q}(\mu_e^c - \mu_e), \quad (A2)$$

where $n_{n,i} = \left(\frac{\partial P_i}{\partial \mu_n} \right)_{\mu_n^c, \mu_e^c}$ is the baryon number density in each of the two phases, and $n_{e,i} = \left(\frac{\partial P_i}{\partial \mu_e} \right)_{\mu_n^c, \mu_e^c}$ the corresponding electron number density.

The pressure difference becomes

$$\Delta P = (n_{n,q} - n_{n,h})(\mu_n^c - \mu_n) + (n_{e,q} - n_{e,h})(\mu_e^c - \mu_e), \quad (A3)$$

where the last term vanishes because of the homogeneous electron distribution.

The surface tension is expanded as

$$\sigma(\mu_n, \mu_e) \simeq \sigma(\mu_n^c, \mu_e^c) - \sigma'_n(\mu_n^c - \mu_n) - \sigma'_e(\mu_e^c - \mu_e), \quad (\text{A4})$$

where σ'_i is the partial derivative with respect to μ_i . Since $\mu_e \ll \mu_n$ we may ignore the third term compared to the second term.

Combining Eqs. (A3) and (A4) with the Laplace condition for mechanical (meta)stability,

$$P_h - P_q = \frac{(d-1)\sigma(\mu_n, \mu_e)}{r}, \quad (\text{A5})$$

generalized to also include cylindrical and planar geometries give a form for the surface tension, Eqs. (7) and (8), similar to the one in Ref. [28].

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